SECTION A

1. a) Given that the function $f(x) = x^2 + px$ is increasing on the interval $[-1,1]$, find one possible value for p.
b) State with justification whether this is the only possible value for p

2. By completing the square, show that the solutions to the equation $x^2 + 2bx + c = 0$ are given by the formula $x = -b \pm \sqrt{b^2 - c}$

3. a) Where is Hurn?
b) Where is Leuchars?
c) Would you expect it to be windier in Hurn or Leuchars?
d) Here is the daily mean windspeed for Hurn and Leuchars in May 2015

	May 1, 2015	May 2, 2015	May 3, 2015	May 4, 2015	May 5, 2015	May 6, 2015	May 7, 2015	May 8, 2015	May 9, 2015	May 10, 2015
Hurn	9	11	14	8	19	17	8	8	12	7
Leuchars	8	13	15	10	12	7	11	7	8	11
	May 11, 2015	May 12, 2015	May 13, 2015	May 14, 2015	May 15, 2015	May 16, 2015	May 17, 2015	May 18, 2015	May 19, 2015	May 20, 2015
Hurn	7	8	4	8	7	11	7	14	11	10
Leuchars	18	20	10	8	11	16	16	8	9	10
	May 21, 2015	May 22, 2015	May 23, 2015	May 24, 2015	May 25, 2015	May 26, 2015	May 27, 2015	May 28, 2015	May 29, 2015	May 30, 2015
Hurn	6	5	7	6	6	6	7	10	11	7
Leuchars	16	13	8	14	8	10	10	15	9	11

Investigate which town is windier.

4. A car of mass 1200 kg is moving along a level road. The car's engine provides a constant driving force. The motion of the car is opposed by a constant resistance. Given that the car is accelerating at $2\ ms^{-1}$ and that the magnitude of the driving force is three times the magnitude of the resistance force, show that the magnitude of the driving force is 3600 N.

5. A uniform ladder of mass 20 kg and length 8 m rests against a smooth vertical wall with its lower end on rough horizontal ground. The coefficient of friction between the ground and the ladder is 0.3. The ladder is inclined at an angle θ to the horizontal, where $\tan\theta = 2$. A boy of mass 30 kg climbs up the ladder. By modelling the ladder as a uniform rod, the boy as a particle and the wall as smooth and vertical,

a) find how far up the ladder the boy can climb before the ladder slips
b) Criticise this model with respect to
   i) the ladder ii) the wall

6. Without using your calculator, find the exact values of

a) $2\sin 22.5° \cos 22.5°$ b) $2\cos^2 15° - 1$

c) $(\sin 75° - \cos 75°)^2$ d) $\frac{2\tan\frac{\pi}{8}}{1-\tan^2\frac{\pi}{8}}$

SECTION B

1. Work these out without using a calculator. Give your answers in terms of π

a) arc cos (0) b) arc sin (1) c) arc tan (-1) d) arc sin (- $\frac{1}{2}$)

e) arc cos $(-\frac{1}{\sqrt{2}})$ f) arc tan $(-\frac{1}{\sqrt{3}})$ g) arc sin (sin $(\frac{\pi}{3})$) h) arc sin (sin $(\frac{2\pi}{3})$)

2. a) By expanding $sin(3x + 2x)$ and $sin(3x - 2x)$, show that $\sin 5x + \sin x \equiv 2 \sin 3x \cos 2x$

b) Hence find $\int \sin 3x \cos 2x \, dx$

3. The masses of arctic foxes are found and the mean mass was 6.1 kg. The variance was 4.2
An outlier is an observation which lies ± 2 standard deviations from the mean.

a) Which of these arctic fox masses are outliers?
2.4 kg 10.1 kg 3.7 kg 11.5 kg

b) What are the smallest and largest masses that an arctic fox can be without being an outlier?

4. A particle P is moving on the x-axis with constant deceleration $4\ ms^{-2}$. At time t = 0, P passes through the origin O with velocity 14 ms^{-1} in the positive direction. The point A lies on the axis and OA = 22.5 m. Find

a) the difference between the times when P passes through A
b) the total distance travelled by P during the interval between these times.

5. The lifetime, x, in hours of 70 light bulbs is shown in the table.

Lifetime, (x hours)	$20 < x \leq 22$	$22 < x \leq 24$	$24 < x \leq 26$	$26 < x \leq 28$	$28 < x \leq 30$
Frequency	3	12	40	10	5

The data is coded using $y = \frac{x-1}{20}$

a) Estimate the mean of the coded values $\bar{y}$
b) Hence find an estimate for the mean lifetime of the lightbulbs, $\bar{x}$
c) Estimate the standard deviation of the lifetimes of the bulbs.

6. a) Show that the equation $3 \sin^2 x - \cos^2 x = 2$ can be written as $4 \sin^2 x = 3$

b) Hence solve the equation $3 \sin^2 x - \cos^2 x = 2$ in the interval $-180° \leq x \leq 180°$, giving your answer to 1 decimal place

SECTION C

1. Given that $\int_0^{\theta} 5\tan x \sec^4 x = \frac{15}{4}$ $where$ $0 < \theta < \frac{\pi}{2}$, find the exact value of θ

2. A curve C is described by the equation $2x^2 + 3y^2 - x + 6xy + 5 = 0$
Find an equation of the tangent to C at the point (1,-2), giving your answer in the form $ax + by + c = 0$, where *a, b* and *c* are integers.

3. A sector of a circle of radius 28 cm has perimeter P cm and area A cm^2. Given that A = 4P, find the value of P.

4. A pebble is projected vertically upwards with speed $21\ ms^{-1}$ from a point $32\ m$ above the ground. Find

a) the speed with which the pebble hits the ground
b) the total time for which the pebble is more than 40 m above the ground.
c) Sketch a velocity-time graph for the motion of the pebble from the instant it is projected to the instant it hits the ground, showing the values of *t* at any points where the graph intercepts the horizontal axis.
d)

5. A train, travelling on a straight track, is slowing down with constant deceleration $0.6\ ms^{-2}$. The train passes one signal with speed $72\ km\ h^{-1}$ and a second signal 25 s later. Find:
a) the velocity in $km\ h^{-1}$, of the train as it passes the second signal.
b) the distance between the signals.

6. $f(x) = \frac{9x^2+4}{9x^2-4}$, $\quad x \neq \pm\frac{2}{3}$

a) Given that $f(x) = A + \frac{B}{3x-2} + \frac{C}{3x+2}$, find the values of the constant A, B and C.
b) Hence find the exact value of $\int_{-\frac{1}{3}}^{\frac{1}{3}} \frac{9x^2+4}{9x^2-4}$ dx, writing your answer in the form a + b ln c, where a,b and c are rational numbers to be found.

ANSWERS
SECTION A

1 a) $p \geq 2$ b) No. Can be any $p \geq 2$

2) Proof

3. a) Dorset in Southern England b) On the North-East coast of Scotland
c) Your opinion…
d) You must give some mathematical analysis to your argument. Some statistical calculations and diagrams are essential.

4. Proof

5. a) $5\frac{1}{3}\ m$
b) i) the ladder may not be uniform
ii) there would be friction between the ladder and the vertical wall

6. a) $\frac{\sqrt{2}}{2}$ b) $\frac{\sqrt{3}}{2}$ c) $\frac{1}{2}$ d) 1

SECTION B

1) a) $\frac{\pi}{2}$ b) $\frac{\pi}{2}$ c) $-\frac{\pi}{4}$ d) $-\frac{\pi}{6}$ e) $\frac{3\pi}{4}$ f) $-\frac{\pi}{6}$ g) $\frac{\pi}{3}$ h) $\frac{\pi}{3}$

2) a) proof b) $-\frac{1}{10}\cos 5x\ -\frac{1}{2}\cos x + c$

3 a) 11.5 kg b) Smallest 2.0 kg, Largest 10.2 kg

4 a) 2 s b) 4 m

5 a) 1.2 hours b) 25.1 hours c) 1.76 hours

6.a) Proof b) $-120°,\ -60°,\ 60°,\ 120°$

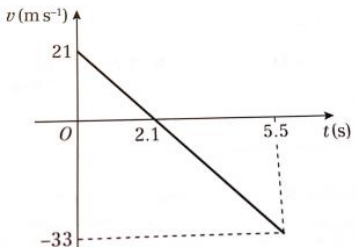
SECTION C

1.$\frac{\pi}{4}$

2. $3x + 2y + 1 = 0$

3. 78.4

4. a) $33\ ms^{-1}$ b) 3.4 s (2 s.f.) c)



5. a) 18 $km\ h^{-1}$ b) 312.5 m

6. a) $A = 1,\ B = 2,\ C = -2$
b) $a = 4,\ b = \frac{2}{3}\ ,\ c = \frac{1}{4}$